

# EOSC 512

## Advanced Geophysical Fluid Dynamics

Fall 2016 (Revised Nov 20 2016)

### Course Description

The purpose of this course is to a) introduce the student to the dynamical principles governing the large-scale low-frequency motions in strongly rotating fluid systems (like the ocean, atmosphere, and liquid planetary core), and their consequences, and b) to develop the skills required to manipulate and use these equations to solve problems.

### Prerequisites

Formally none. However, this course is mathematical and assumes a working knowledge of vector calculus (e.g. div, grad, curl), partial differential equations (i.e. you can solve at least some of them), and some exposure to complex analysis (e.g. you know that if  $z = x + iy$ , then  $e^z = e^x \cos(y) + ie^x \sin(y)$ ). A background in fluid dynamics, geophysics, atmospheric sciences, and/or oceanography is not required (although undoubtedly will be helpful).

### Instructor

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### Meeting Times

Lectures: Tuesday and Thursday 9:30-11a, EOS-Main Room 101.

### Class Website

I will post copies of my lecture notes, problem sets and problem set solutions at <https://www.eoas.ubc.ca/~swaterma/512/index.html>.

### Course Learning Goals

The purpose of this course is to a) introduce the student to the dynamical principles governing the large-scale low-frequency motions in strongly rotating fluid systems (like the ocean, atmosphere, and liquid

planetary core), and their consequences, and b) to develop the skills required to manipulate and use these equations to solve problems.

At the end of this course, students should be able to:

1. write down the ‘standard equations’ of geophysical fluid dynamics (GFD), identify the different terms, evaluate their relative importance based on scaling arguments, and explain how different dynamical features depend on these terms. Examples include the geostrophic and quasi-geostrophic equations, boundary layer equations, and thermodynamic relationships.
2. define standard terms and concepts used in GFD (the “language” of GFD), and identify them when they arise in the context of dynamical interpretations. Examples include Eulerian, Lagrangian, hydrostatic, Boussinesq, the Coriolis acceleration/force, Ekman layers, vorticity, geostrophic, barotropic, and baroclinic.
3. use standard mathematical techniques to simplify complex equation sets relevant to GFD. Examples include linearization, scaling arguments, normal mode techniques, and complex exponentials in wave and instability problems.
4. use the appropriate approximations and mathematical techniques to simplify and solve particular “canonical” GFD problems. Examples include a description of Taylor columns, a description of Ekman layers, spin-down problems, Rossby adjustment problems, wave problems in non-rotating and rotating systems, and instability problems.

## Course Outline

### 1. Some Basics

- Definition of a fluid
- The continuum hypothesis
- The fluid element
- Kinematics
  - The Eulerian approach
  - The Lagrangian approach
  - The fluid trajectory
- Rates of change
- Streamlines & streak lines

### 2. The Continuum Equations

- The conservation of mass
- The momentum equation
- Diversion: The stress tensor
- The momentum equation in differential form

### 3. The Stress Tensor for a Fluid & the Navier-Stokes Equations

- The symmetry of the stress tensor
- Putting the stress tensor in diagonal form
- The static pressure
- Analysis of fluid motion at a point
- The vorticity
- The rate of strain tensor
- Principal strain axes & the decomposition of the motion

- The relation between stress and rate of strain
  - The coefficient of viscosity  $\mu$
  - The Navier-Stokes equations
4. **The Equations of Motion in a Rotating Coordinate System**
- Rates of change of vectors to a fixed vs. rotating observer
  - The Coriolis acceleration/force
  - The Navier-Stokes equations in a rotating frame
5. **Boundary Conditions & Frictional Boundary Layers**
- Boundary conditions at a solid surface
  - Boundary conditions at a fluid surface
  - Ekman layers over a solid surface
  - Ekman layers below a free surface (Nansen’s problem)
  - Example applications to coastal upwelling and the ocean’s gyre circulations
  - The Ekman spin-down time
6. **Fundamental Theorems: Vorticity & Circulation**
- Kelvin’s circulation theorem
  - Example application: The Rossby wave
  - Further consequences of Kelvin’s theorem: Vortex tube stretching
  - The vorticity equation
  - Example application: Thermal wind
  - Example application: The Taylor-Proudman theorem
  - Ertel’s theorem and the definition of potential vorticity
7. **Geostrophy, Quasi-Geostrophy & the Quasi-Geostrophic Potential Vorticity Equation**
- Synoptic-scale dynamics: Geostrophy scaling & geostrophic balance
  - The quasi-geostrophic potential vorticity equation
  - Example application: Baroclinic Rossby waves
8. **A (very brief) Overview of Waves, Instability & Turbulence**

## Texts

I will post copies of my own class notes after class at <https://www.eoas.ubc.ca/~swaterma/512/index.html>. These should act as a supplement to your own notes taken during class.

The class does not follow any specific text, but the texts listed below can be helpful for additional reading. Physical copies of all of texts are available for a shortened, in-library loan period via Course Reserves at the Woodward Library. Items marked with an \* are also available online as e-books through the UBC Library website.

1. \*Kundu, P.K. *Fluid Mechanics*. Academic Press. (Any edition 1990 or later.)
2. Batchelor, G.K. *An Introduction to Fluid Dynamics*. Cambridge Univ. Press. (Any edition 1967 or later.)
3. \*Aris, R. *Vectors, Tensors and the basic equations of fluid mechanics*. Prentice Hall. (Any edition 1962 or later.)
4. Cushman-Roisin, B. *Introduction to Geophysical Fluid Dynamics*. Prentice-Hall. (Any edition 1994 or later.)

5. Gill, A.E., *Atmospheric-Ocean Dynamics*. Academic Press. 1982.
6. \*Pedlosky, J. *Geophysical Fluid Dynamics*. Springer Verlag. (Any edition 1979 or later.)
7. \*Marshall, J. and R.A. Plumb. *Atmosphere, Ocean and Climate Dynamics: An Introductory Text*. Academic Press. 2008.

## Evaluation

### 1. Problem Sets (50%)

I place a lot of value in applying the concepts we are learning to the exercise of solving problems. As such, a problem set consisting of 1-3 problems will be assigned weekly on Fridays. It will be available online by Friday afternoon at <https://www.eoas.ubc.ca/~swaterma/512/index.htm>. It will be due 1 week later (feel free to hand-in in class on Thursday or in my mailbox in the EOAS main office by Friday end-of-day). Problem sets will be graded on a 1 point per question basis, with the possibility for partial credit or extra merit via scores of 1- and 1+. I encourage you to discuss the problems with your peers. Please however write-up your problem solutions individually. Problem solutions will be posted online also at <https://www.eoas.ubc.ca/~swaterma/512/index.htm> on Mondays. I obviously can't accept problem sets after this date. I'll throw away your problem set with the lowest grade when computing the final marks, to allow for things that inevitably come up that causes us to miss deadlines from time-to-time. Please review these solutions and come speak to me if you have questions about the problems remaining.

### 2. Final Exam (50%)

The final exam will be held **in class** on **Thursday November 24** (the second last week of class) from **9a - 12p**. Please reserve this time in your schedules and see me now if this time will be problematic for you. The exam will be in a so-called 2-stage format. The first 2 hours will be spent working on problems as an individual. This work will then be handed in. In the final hour, you will work together in small groups on the same set of questions. This work will then be handed in as a group. Your final grade will be made up of 85% of your individual score and 15% of your group score, unless the group score is lower than your individual score (happens rarely in practice). In this case, your final grade will be made up of your individual exam grade only.

The two-stage exam format was first introduced in the UBC Faculty of Science in 2009, and is now being used in at least 20 science courses. It allows students to receive immediate, targeted feedback on their solutions from their fellow students, and to see alternative approaches to problems. Students' response to the use of two-stage exams is in general overwhelmingly positive, although I am interested in your own feedback. More information about the format can be found here: [http://www.cwsei.ubc.ca/resources/files/Two-stage\\_Exams.pdf](http://www.cwsei.ubc.ca/resources/files/Two-stage_Exams.pdf).

## Important Dates

Date	Event
Thu Sep 8	First class at 9:30a
Tue Sep 20	Last day to withdraw from course without a 'W' appearing on transcript
Fri Sep 23	First problem set due by end of the day in my mailbox (subsequent problem sets are due on Fridays by end-of-day)
Thu Oct 13	No class!
Tue Nov 20	Last class: Review and pre-exam Q & A
Thu Nov 24	Final exam 9a - 12p

## Final Note

This is the my first time teaching this course, so there may be some on-the-fly learning about what works and what doesn't as we go along. In particular, I expect the topic list to evolve as we go. Comments on anything to do with the course (content, format, pace, level, lecture style, problem sets, *etc.*) are always very welcome. There will be a formal course evaluation at the end of the term, but if you tell me about any issues/suggestions earlier, we can try to make improvements this term as well.